Theorems justifying steps in algebraic transformations

University of ………………….. (if possible, change icon accordingly)

Mathematics is considered a difficult subject. This questionnaire is part of a research on basic principles and reasons for difficulties in mathematics education. Thus you are just asked basic knowledge, most of which will remind you of early mathematics classes.

1) Below you find basic laws of algebra (i.e theorems). (a) Do you remember some laws? (b) Do you even remember the names of some laws? (c) Can you apply the laws to numbers?

   a) \( a + b = b + a \)  
      (a) X yes no  (b) law of commutativity for +
      (c) \( 2 + 3 = 3 + 2 \) … \( 5 = 5 \)

   b) \( a \cdot b = b \cdot a \)  
      yes no  law ……………………………………….
      \( \ldots = \ldots = \ldots = \ldots \)

   c) \((a + b) + c = a + (b + c)\)  
      yes no  law ……………………………………….
      \( \ldots = \ldots = \ldots = \ldots \)

   d) \( a \cdot (b + c) = a \cdot b + a \cdot c \)  
      yes no  law ……………………………………….
      \( \ldots = \ldots = \ldots = \ldots \)

   e) \( a \cdot 1 = a \)  
      yes no  law ……………………………………….
      \( \ldots = \ldots = \ldots \)

   f) \( a + 0 = a \)  
      yes no  law ……………………………………….
      \( \ldots = \ldots = \ldots \)

   g) \( a - a = 0 \)  
      yes no  law ……………………………………….
      \( \ldots = \ldots = \ldots \)

2) Do you remember any other laws of algebra, not yet mentioned above?

   a) ……………………………………….  
      law ……………………………………….

   b) ……………………………………….  
      law ……………………………………….

   c) ……………………………………….  
      law ……………………………………….

3) Simplify the following algebraic expressions, please. Simplifying such expressions is learned together with laws of algebra; but usually one simplifies without laws, for instance:

   a) \( 2 \cdot (x + 3 \cdot y) - 6y = 2 \cdot x + 6 \cdot y - 6 \cdot y = 2 \cdot x \)

   b) \( 2 \cdot (x + 3 \cdot y) + 6y = \) ………………………………………. = …

   c) \( r \cdot r \cdot (2 + s) = \) ………………………………………. = …

   d) \( (u + 1) \cdot (u - 1) = \) ………………………………………. = …

   e) \( (x + y) \cdot (x - y) = \) ………………………………………. = …
This page is about using laws to justify steps in simplifications. We give the following abbreviations for laws:

\[
\begin{align*}
[C+] & \quad a + b = b + a \\
[A+] & \quad (a + b) + c = a + (b + c) \\
[A–] & \quad (a + b) - c = a + (b - c) \\
[U+] & \quad a + 0 = a \\
[D+] & \quad a(b + c) = ab + ac \\
[I+] & \quad a - a = 0 \\
[U·] & \quad a \cdot 1 = a \\
[D–] & \quad a(b - c) = a \cdot b - a \cdot c
\end{align*}
\]

Here is an example of stepwise justifying a simplification by use of these laws and by calculating natural numbers \([N+ – ·]\):

\[
2 \cdot (x + 3 \cdot y) - 6 \cdot y \quad \Rightarrow (2 \cdot x + 2 \cdot (3 \cdot y)) - 6 \cdot y \quad \Rightarrow (2 \cdot x + (2 \cdot 3) \cdot y) - 6 \cdot y
\]

\[
\Rightarrow (2 \cdot x + 6 \cdot y) - 6 \cdot y \quad \Rightarrow 2 \cdot x + (6 \cdot y - 6 \cdot y) \quad \Rightarrow 2 \cdot x + 0 \quad \Rightarrow 2 \cdot x
\]

4) **Similarly describe a stepwise justification of the following simplifications, please;**

Take as many steps you need:

a) \[2 \cdot (x + 3 \cdot y) + 6 \cdot y \]

\[
\begin{align*}
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ]
\end{align*}
\]

b) \[r + r \cdot (2 + s) \]

\[
\begin{align*}
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ]
\end{align*}
\]

c) \[(u + 1) \cdot (u - 1) \]

\[
\begin{align*}
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ] \\
= \quad & \ [ \ldots \ ]
\end{align*}
\]

5) **Can the simplification \((x+y) \cdot (x-y) = x \cdot x - y \cdot y\) be justified using the above laws only?**

a) If “yes”, give the first three steps and justifications, please:

\[
(x+y)(x-y) \Rightarrow \ [ \ldots \ ] \Rightarrow \ [ \ldots \ ] \Rightarrow \ [ \ldots \ ]
\]

b) If “no”, give some missing laws, please:

\[
\begin{align*}
\text{law} & \quad \ [ \ldots \ ] \\
\text{law} & \quad \ [ \ldots \ ] \\
\text{law} & \quad \ [ \ldots \ ]
\end{align*}
\]

Thank you for attention! For results see www.ist.tugraz.at/projects/isac.