Initiative for an EU-wide data collection by questionnaire  
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**What is Mathematics for Students at which Level:**  
a Natural or a Formal Language ?

Background information:

European education faces “alarming decline in young people's interest for key science studies and mathematics” [1]. This paper acts on the assumption, that the difference in teaching mathematics between schools and academia is one important reason for the “alarming decline”. And with respect to longterm oscillations in pedagogy and Bourbaki's influence, Pierre Cartier says [2] “a consequence of the evident excess of abstraction in pedagogy is that nowadays the balance has moved to the other side and the idea of what a proof means is not considered important”. The development of educational math software seems to reflect the latter opinion: there are lots of products available for the phases of modelling and interpretation, but none (as “black boxes” Computer Algebra Systems are none!) for the formal phase of problem solving.

With this state of educational affairs in mind, a questionnaire on “theorems justifying steps in simple algebraic transformations” will be handed out to 500 students from Austria, Finland, Germany, Great Britain, Serbia and Spain in different educational institutions on the secondary (high schools) and tertiary level (polytechnic and universities). The results of pre-tests with this questionnaire are surprisingly clear:

Students know theorems, but they cannot use them !

This conclusion comes from comparing the first and the second page of the questionnaire: The first page asks for theorems required for algebraic simplification, and most students know them; also the simplification of algebraic terms is done correctly by most of the students.

The second page asks to use the same theorems for stepwise justification of the simplifications, most of which the students had accomplished on the first page. But here the result is: Most of the students have no idea of how to use theorems for stepwise justifying simplifications.

The phenomenon addressed by the questionnaire pinpoints a key issue at the interface between school math and academic math: while school teachers have good reasons from developmental psychology not to emphasize the competency requested on the second page of the questionnaire, academic math must rely on this competency, because it is a prerequisite for mathematical proof, and for formal mathematics in general.

EU-wide data collection with this questionnaire shall contribute to research on math education at the interface between school and academia. The following points try to identify some starting points and hypothesis for research:

1. The theory of language acquisition is appropriate to explain the phenomena revealed by the questionnaire. The last but one didactic innovation, the “new math” in the seventies of the last century, heavily relied on research contributed by Jean Piaget [3]. Piaget also developed a theory of language, but we assume, that this theory has found too little attention in didactics of mathematics. Even with this in mind we assume, that further theories of language acquisition need to be considered for explaining the phenomena revealed by the questionnaire.
2. The language of mathematics is a **written language**. Thus just the part of the theory of language acquisition will be appropriate for this research, which goes beyond phonetic aspects. The parts concerned with syntax and semantics might well stem from research on instruction in Latin: Translating a Latin writer starts with searching the predicate – by looking at the endings of the words (at syntax, i.e. formal structures). Next follows the search for the subject, and again this search is guided by looking at the endings. And after all that, gradually the semantics comes in, the dictionary is asked for the meaning of the words identified important by grammatical structures.

3. The notion of **“reflection”** is particularly promising to investigate the phenomena. Thus the theory we need must address the reflective nature of language: Our probands are able to handle formulas (on the first page of the questionnaire), but they cannot use formulas to justify formulas. As teaching experience shows, students are just not interested in “justifying formulas by formulas”; this indicates, that mental maturity is not ready for such interest. But just that is the essence of mathematical proof! A well-know and illuminative example are Gödel's theorems and their proof by the so-called "Gödel numberings"; the trick of these numberings do nothing else than relating formulas (designating a certain formal theory) to formulas of a meta theory (a formal theory about the theory under consideration).

4. The reflective nature of language **can “not be taught”**: There is no sense to drill students with exercises shown on the second page of the questionnaire, although it would be possible with defensible effort. Such a training would not immediately help a student to understand Gödel's theorems; rather the understanding of formal systems (and thus accepting to “justify formulas by formulas”) is up to processes of mental maturing. Such processes can be prepared and fostered, but cannot be immediately enforced by teaching some learning unit.

5. Since familiarity with the reflective nature of mathematics depends on mental maturing (and cannot be the content of a learning unit), approaches towards essential insights about “justifying formulas by formulas” must be **at an individual pace**. Such approaches are to be supported by software [4], which provides a steady offer to learner, to have a look at the formal/reflective nature of math, while their math lessons can be accomplished without these insights.

From such research we expect massive impact on the practice of math education at the interface between schools and academia.

Whether there could be immediate **consequences for the practice of math education** will be seen from feedback given by the colleagues who collect the data from their students.

References:


[3] TODO