On Analogies between Geometry and Algebra on Computers

Possibilities for Joint Development GeoGebra – ISAC?

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Outline

1. Analogogies in CAS and DGS
   - Analogies: Steps in Geometry – Steps in Algebra
   - A Program Drawing an Ortho-Center
   - A Program-Interpreter for “Guided Interaction”

2. Demo: Guided Interaction in Algebra

3. Joint Development GeoGebra – ISAC?
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A step propagates a Calculation/Construction by adding a new Term/geomtric object (GeO)

\[
\text{step} : \text{Context} \times \text{State} \times Pst \times \text{Interact} \rightarrow \text{Context} \times \text{State} \times Pst \times \text{Term} \\
\text{step} : \text{Context} \times \text{State} \times Pst \times \text{Interact} \rightarrow \text{Context} \times \text{State} \times Pst \times \text{GeO}
\]

where

\( Pst = Program \times (\text{Set of Locations}) \)

\( \text{Interact} = \)

1. **Next** by system
2. **Rule** by user
3. **Term** by user

\( \) in Calculation \( \) in Construction

Walther Neuper
Models of Mathematics
Three Kinds of Steps in Geometry – in Algebra

A **step** propagates a Calculation/Construction by adding a new Term/geometric object (*GeO*)

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\text{step} : \text{Context} \times \text{State} \times Pst \times \text{Interact} \longrightarrow \text{Context} \times \text{State} \times Pst \times \text{Term} \\
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\text{Interact} = \\
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These programs *stepwise* create/check the construction . . .

program OrthoCenter P_1 P_2 P_3 =
M = let (l_1 = Bisector P_1 P_2 and l_2 = Bisector P_2 P_3)
or
   (l_1 = Bisector P_1 P_2 and l_2 = Bisector P_1 P_3)
or
   (l_1 = Bisector P_1 P_3 and l_2 = Bisector P_2 P_3)
in intersect_2 l_1 l_2

program Bisector P_1 P_2 =
l = let c_1 = Circle (P_1, Distance P_1 P_2);
c_2 = Circle (P_2, Distance P_1 P_2);
(Q_1, Q_2) = intersect_1 c_1 c_2
in make_line Q_1 Q_2

... without any input-/output statements ???.
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Automatically Generated “Guided Interaction”

- input/output done by **side-effects** of (functional !) program
- side-effects handled by a **program-interpreter**
- the program-interpreter **guarantees correctness** of steps
- program-interpreter serves a dialog-guide
- we have **separation of concerns**:
  - math-programmer concentrates on math (no in/output)
  - dialog-programmer is not concerned with math details
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Possibilities for Joint Development GeoGebra – *ISAC*

1. Share **language** for guided user-interaction?
   - + module for user-guidance?
   - + user model?
   - + learning theory (knowledge space theory, …)?

2. Integrate **whole** *ISAC*?

3. Share *ISAC*’s **algebra-engine** (instead of present CAS)?